Journal of Engineering Physics and Thermophysics, Vol. 80, No. 4, 2007

CALCULATION OF STRESSES AND STRAINS IN A SPHERICAL VOLUME FILLED WITH WATER CAUSED BY ITS FREEZING

V. A. Sychevskii UDC 519.6:539:536

Formulas for calculating stresses and strains in a spherical water–ice system and a numerical method for solving the Stefan problem are presented. The processes of freezing and development of stresses and strains in such a system have been calculated.

Introduction. The last few years have seen a rising interest in nontraditional methods of material processing. One of them is freezing. It is used to preserve perishable food products, to separate moisture in concentrating liquid substances, and in sublimation drying of bodies. Industrial production of foodstuffs by the sublimation method has found wide application and continues to develop intensively. Sublimation drying is an effective method of dehydration without affecting the structure, shape, and volume of materials with an unstable structure and a strong dependence of dimensions on the moisture content in them.

Freezing of cells and individual organs is currently an urgent problem in medicine. It is used in prolonged storage, transportation, and transplantation of organs, in creating banks of donor sperm and ova, and in storing skin, cornea, and bone marrow. One of the reasons inhibiting the application of freezing in medicine is mechanical destruction of bodies.

Freezing of organic objects is topical in agriculture, where the problem of the influence of low temperatures in winter and during light frosts on crops, perennial plants, and ripe crops is acute. There exists the problem of freezing of inanimate objects, among which are buildings, structures, grounds and soils. For instance, one reason for the destruction of buildings and engineering structures is the freezing of water in the pores and voids. The cyclicity of this process leads to the formation of a network of microcracks, which decreases the strength and the bearing capacity of structures and leads, in the course of time, to their destruction. Cases are known where heaving of the ground due to its freezing led to a strong deformation and destruction of buildings.

Formulation of the Problem and Methods of Its Solution. To investigate the development of stresses and strains in a material caused by its freezing, let us consider the process of freezing of a liquid confined in a spherical shell representing a conglomerate of ice and the structurizing base (material). Consider two variants of freezing out of water: in a rigid shell and in a shell that can deform under pressure in an expanding water–ice system.

When the liquid is confined in a rigid shell and freezes out, then the ice formed (i.e., ice I), expanding, compresses the water and a complex strain state arises in the system. The larger the ice interlayer, the higher the pressure in the liquid phase and finally it reaches a value equal to $P_{cr} = 2.1 \cdot 10^8$ Pa at which ice III is formed. The calculations show that the water density at P_{cr} has a value of $1.11 \cdot 10^3$ kg/m^3, whereas the density of ice III is in the range from $1.1 \cdot 10^3$ to $1.15 \cdot 10^3$ kg/m^3. Therefore, it may be considered, with an error of less than 5%, that they are in agreement, and we may assume them to be equal. This means that freezing will not lead to the formation of other modifications of ice except for ice III and will not increase the pressure in the system to above P_{cr}, which considerably simplifies the problem.

Let us describe the stress-strain state upon freezing of the water–ice system confined in a shell representing a conglomeration of ice and a structurizing base (material). Suppose that the mechanical processes proceed much faster than the thermal ones and the system always has time to react to the temperature changes in it. Therefore, we assume

---

A. V. Luikov Heat and Mass Transfer Institute, National Academy of Sciences of Belarus, 15 P. Brovka Str., Minsk, 220072, Belarus. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 80, No. 4, pp. 173–180, July–August, 2007. Original article submitted December 29, 2005; revision submitted October 2, 2006.

 1062-0125/07/8004-0820

that the material is in static equilibrium and, consequently, the mechanical problem with regard for the spherical symmetry will take the form [1]

$$\frac{\partial}{\partial r}\left(\frac{1}{r^2}\frac{\partial r^2 u}{\partial r}\right)=0\,. \tag{1}$$

The boundary layer for each ice interlayer in the case where the outer shell is rigid and does not deform and in the liquid phase the hydrostatic pressure $P_{\rm w}$ is maintained is of the form

$$r=r_{\rm out}:\ \ u=0\,;\ \ r=r_{\rm ice0}:\ \ \sigma_{rr}=P_{\rm w}\,. \tag{2}$$

If, on the inner surface of the outer shell, the pressure $P_{\rm ex}$ is applied, then

$$r=r_{\rm out}:\ \ \sigma_{rr}=P_{\rm ex}\,;\ \ r=r_{\rm ice0}:\ \ \sigma_{rr}=P_{\rm w}\ \ (P_{\rm ex}<0)\,. \tag{3}$$

The solution of (1) has the form (see, e.g., [1])

$$u=ar+\frac{b}{r^2}\,. \tag{4}$$

The strain and stress tensors are calculated by the formulas

$$\varepsilon_{rr}=a-\frac{2b}{r^3}\,,\ \ \varepsilon_{\theta\theta}=\varepsilon_{\varphi\varphi}=1+\frac{b}{r^3}\,, \tag{5}$$

$$\sigma_{rr}=\frac{E_{\rm ice}}{1-2\nu}\,a-\frac{2E_{\rm ice}}{1+\nu}\frac{b}{r^3}\,,\ \ \sigma_{\theta\theta}=\sigma_{\varphi\varphi}=\frac{E_{\rm ice}}{1-2\nu}\,a+\frac{E_{\rm ice}}{1+\nu}\frac{b}{r^3}\,. \tag{6}$$

Using the boundary condition (2), we represent the constants a and b as

$$a=-\frac{b}{r_{\rm out}^3}\,,\ \ b=-\frac{P_{\rm w}}{E_{\rm ice}}\frac{r_{\rm ice0}^3 r_{\rm out}^3}{\frac{1}{1-2\nu}\,r_{\rm ice0}^3+\frac{2}{1+\nu}\,r_{\rm out}^3}\,. \tag{7}$$

In this case, the stress tensor components will be

$$\sigma_{rr}=P_{\rm w}\frac{r_{\rm ice0}^3 r_{\rm out}^3}{\frac{1}{1-2\nu}\,r_{\rm ice0}^3+\frac{2}{1+\nu}\,r_{\rm out}^3}\left(\frac{1}{1-2\nu}\frac{1}{r_{\rm out}^3}+\frac{2}{1+\nu}\frac{1}{r^3}\right), \tag{8}$$

$$\sigma_{\theta\theta}=\sigma_{\varphi\varphi}=-P_{\rm w}\frac{r_{\rm ice0}^3 r_{\rm out}^3}{\frac{1}{1-2\nu}\,r_{\rm ice0}^3+\frac{2}{1+\nu}\,r_{\rm out}^3}\left(\frac{1}{1+\nu}\frac{1}{r^3}-\frac{1}{1-2\nu}\frac{1}{r_{\rm out}^3}\right). \tag{9}$$

From the last formula it is seen that $\sigma_{\theta\theta}$ and $\sigma_{\varphi\varphi}$ can change sign: at $r=\sqrt[3]{(1-2\nu)/(1+\nu)}\;r_{\rm out}$, $\sigma_{\theta\theta}=\sigma_{\varphi\varphi}=0$, at $r>\sqrt[3]{(1-2\nu)/(1+\nu)}\;r_{\rm out}$, $\sigma_{\theta\theta}=\sigma_{\varphi\varphi}<0$, and at $r<\sqrt[3]{(1-2\nu)/(1+\nu)}\;r_{\rm out}$, $\sigma_{\theta\theta}=\sigma_{\varphi\varphi}>0$.

For the boundary conditions (3), the constants a and b will be

$$a = -\frac{1-2\nu}{E_{ice}} \frac{P_w r_{ice0}^3 - P_{ex} r_{out}^3}{r_{out}^3 - r_{ice0}^3}, \tag{10}$$

$$b = -\frac{(1+\nu)(P_w - P_{ex})}{2E_{ice}} \frac{r_{ice0}^3 r_{out}^3}{r_{out}^3 - r_{ice0}^3}. \tag{11}$$

In this case, the stress-tensor components are defined by the expressions

$$\sigma_{rr} = -\frac{P_w r_{ice0}^3 - P_{ex} r_{out}^3}{r_{out}^3 - r_{ice0}^3} + \frac{(P_w - P_{ex})\, r_{ice0}^3 r_{out}^3}{r_{out}^3 - r_{ice0}^3} \frac{1}{r^3}, \tag{12}$$

$$\sigma_{\theta\theta} = \sigma_{\varphi\varphi} = -\frac{P_w r_{ice0}^3 - P_{ex} r_{out}^3}{r_{out}^3 - r_{ice0}^3} - \frac{(P_w - P_{ex})}{2} \frac{r_{ice0}^3 r_{out}^3}{r_{out}^3 - r_{ice0}^3} \frac{1}{r^3}. \tag{13}$$

The hydrostatic pressure P_w of the water is calculated by the formula

$$P_w = K_w \frac{V - V_0}{V_0}. \tag{14}$$

For small strains at $r_0 >> \Delta r$, neglecting the terms of a higher order of smallness, we will have

$$P_w = 3K_w \frac{\Delta r}{r_0}. \tag{15}$$

Knowing P_w, r_{w0}, and r_{ice0}, we can find the values of the strain and stress tensors at any point of r of both the ice interlayer and the water sphere. The water pressure P_w as a result of the expansion of the ice will be calculated using the relation

$$r_{ice0} + u_{ice0} = r_{w0} + \Delta r. \tag{16}$$

For the boundary conditions (2) at the point of $r = r_{ice0}$

$$u_{ice0} = -\frac{P_w}{E_{ice}} \frac{r_{ice0}(r_{out}^3 - r_{ice0}^3)}{\frac{1}{1-2\nu} r_{ice0}^3 + \frac{2}{1+\nu} r_{out}^3}. \tag{17}$$

Then, using (15) and (16), we obtain

$$P_w = -\frac{r_{w0} - r_{ice0}}{\frac{1}{E_{ice}} \frac{r_{ice0}(r_{out}^3 - r_{ice0}^3)}{\frac{1}{1-2\nu} r_{ice0}^3 + \frac{2}{1+\nu} r_{out}^3} + \frac{1}{K_w} \frac{r_{w0}}{3}}. \tag{18}$$

For the boundary conditions (3) at the point of $r = r_{ice0}$

$$u_{ice0} = -\frac{1-2\nu}{E_{ice}} \frac{P_w r_{ice0}^3 - P_{ex} r_{out}^3}{r_{out}^3 - r_{ice0}^3} r_{ice0} - \frac{(1+\nu)(P_w - P_{ex})}{2E_{ice}} \frac{r_{ice0} r_{out}^3}{r_{out}^3 - r_{ice0}^3}. \tag{19}$$

We calculate the water pressure by the formula

$$P_w = -\frac{(r_{w0} - r_{ice0}) - \frac{3}{2}(1-\nu)\frac{P_{ex} r_{out}^3 r_{ice0}}{E_{ice}(r_{out}^3 - r_{ice0}^3)}}{\frac{1-2\nu}{E_{ice}}\frac{r_{ice0}^4}{r_{out}^3 - r_{ice0}^3} + \frac{1+\nu}{2E_{ice}}\frac{r_{ice0} r_{out}^3}{r_{out}^3 - r_{ice0}^3} + \frac{1}{3}\frac{r_{w0}}{K_w}} . \tag{20}$$

The value of r_{w0} corresponds to the zone of the water–ice phase transitions (r_{ph}) in solving the Stefan problem and is determined by the formula

$$r_{w0} = r_{ph} = \sqrt[3]{\frac{\frac{4}{3}\pi r_{out}^3 \rho_w - M_{ice}}{\frac{4}{3}\pi \rho_w}} . \tag{21}$$

The value of r_{ice0} is determined from the expression

$$r_{ice0} = \sqrt[3]{\left(1 - \frac{\rho_w}{\rho_{iceI}}\right) r_{out}^3 + \frac{\rho_w}{\rho_{iceI}} r_{w0}^3} . \tag{22}$$

Consider now the thermal problem. The heat-conduction equation for the spherically symmetric case has the form

$$c\rho \frac{\partial T}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(\lambda r^2 \frac{\partial T}{\partial r}\right). \tag{23}$$

We write the Stefan condition at the water–ice interface as

$$\lambda_w \frac{\partial T_w}{\partial r}\bigg|_{r=r_{ph}} - \lambda_{ice}\frac{\partial T_{ice}}{\partial r}\bigg|_{r=r_{ph}} = -\rho_w \frac{dr_{ph}}{dt} Q_{ph} , \quad \frac{dr_{ph}}{dt} < 0 . \tag{24}$$

The basic equation describing the water–ice-I phase transition is the Clapeyron–Clausius equation [2, 3]. However, it is difficult to express the pressure dependence of the phase-transition temperature in a wide temperature range by the exact analytical function because of the temperature dependence of the phase-transition heat. Therefore, for further calculations the empirical formula given in [4] was used. In our case, the phase-transition temperature T_{ph} depends on the pressure in the liquid:

$$T_w\big|_{r=r_{ph}} = T_{ice}\big|_{r=r_{ph}} = T_{ph}(P) = 273.16 \sqrt[9]{1 - \frac{P}{395.2}} . \tag{25}$$

On the outer surface of the shell we give the heat transfer by the Newton "law"

$$\lambda \frac{\partial T}{\partial r}\bigg|_{r=R_{out}} = -\alpha (T_s - T_{amb}) . \tag{26}$$

For the center of the sphere, we can write the symmetry condition having the form

$$\frac{\partial T}{\partial r}\bigg|_{r=0} = 0 . \tag{27}$$

The problem was solved numerically using the control-volume approach. Note that coordinate grid numbering is carried out from the center to the outer surface of the body. Some control volumes on the side of the outer surface are allotted to the shell. The general difference scheme of Eq. (23) is given as

$$(c\rho)_i \frac{T_i^{k+1} - T_i^k}{\Delta t} = \frac{1}{r_i^2} \frac{1}{r_{i+1/2} - r_{i-1/2}} \left(\lambda_{i+1/2} r_{i+1/2}^2 \frac{T_{i+1} - T_i}{r_{i+1} - r_i} - \lambda_{i-1/2} r_{i-1/2}^2 \frac{T_i - T_{i-1}}{r_i - r_{i-1}} \right), \tag{28}$$

where $r_{i+1/2} = \frac{r_{i+1} + r_i}{2}$, $r_{i-1/2} = \frac{r_i + r_{i-1}}{2}$. At the sphere center, on the basis of (27), we have the simple relation

$$T_c = T_1 , \tag{29}$$

and on the outer surface we approximate the boundary condition (26) by the difference expression

$$T_s = \frac{T_{i-1} + \mathrm{Bi} \frac{r_i - r_{i-1}}{R_{out}} T_{amb}}{1 + \mathrm{Bi} \frac{r_i - r_{i-1}}{R_{out}}} . \tag{30}$$

Let us now discuss the procedure of calculating the mass of the ice formed and the location and velocity of the crystallization front. Assume that the water goes to ice instantaneously. Using the fact that our system is broken down into control volumes, we write the difference analog of the Stefan condition (24) for the *i*th volume, i.e., we go from the mathematical surface to the finite volume:

$$\lambda_w \frac{T_i - T_{i-1}}{r_i - r_{i-1}} - \lambda_{ice} \frac{T_{i+1} - T_i}{r_{i+1} - r_i} = \frac{\Delta m}{4\pi r_i^2 \Delta t} Q_{ph} . \tag{31}$$

The quantity of the heat removed from the volume is compensated by the delivered heat and the heat of the water–ice phase transition. We calculate the mass of the liquid that has turned to ice at the time step Δt by the formula

$$\Delta m = \left(\lambda_w \frac{T_i - T_{i-1}}{r_i - r_{i-1}} - \lambda_{ice} \frac{T_{i+1} - T_i}{r_{i+1} - r_i} \right) \frac{4\pi r_i^2 \Delta t}{Q_{ph}} . \tag{32}$$

In this case, the temperature T_i of the control volume remains constant and equal to the phase-transition temperature T_{ph}. We subtract the quantity of the frozen-out mass of the liquid Δm from the water mass in the control volume and add it to the ice mass. Now it is easy to calculate the location and velocity of the crystallization front. This proceeds until the whole of the liquid turns to ice. After this, we pass to the next volume into the depth of the material. Obviously, as the size of the control volume and the time step decrease, the accuracy of the calculations will increase. This is the point of the calculation of the water–ice phase transition.

As a result of the discreteness of the numerical scheme, there are certain additional features of its realization. For instance, the temperature T_i in the nodes of control elements varies jumpwise with time from step to step and, consequently, can become lower than the phase-transition temperature T_{ph} even before the process of water–ice transition begins. Therefore, correction of the calculation scheme is conducted as follows. The element temperature is given to be equal to T_{ph}, and the temperature difference $(T_{ph} - T_i)$ is compensated by the water–ice phase transition. In so doing, the quantity of the frozen liquid is calculated by the formula

$$\Delta m = \frac{c_w m_e (T_{ph} - T_i)}{Q_{ph}} . \tag{33}$$

At the end of the process of freezing-out of the liquid in the ith control volume, the mass of the frozen-out water Δm calculated by formula (32) may turn out to be larger than what has really remained. This means that the quantity of heat removed from the element exceeds the quantity of the phase-transition heat in the remaining liquid. To avoid such a situation, the removed heat surplus is covered by cooling the ice to a temperature below the phase-transition temperature T_{ph}:

$$T_i = T_{\text{ph}} - \left(\lambda_{\text{w}} \frac{T_i - T_{i-1}}{r_i - r_{i-1}} - \lambda_{\text{ice}} \frac{T_{i+1} - T_i}{r_{i+1} - r_i} \right) \frac{4\pi r_i^2 \Delta t}{c_{\text{ice}} m_{\text{ice}}} + \frac{m_{\text{e}} Q_{\text{ph}}}{c_{\text{ice}} m_{\text{ice}}} . \tag{34}$$

The computing method above was used by the author to solve various heat- and mass-transfer problems with phase transitions [5–14]: investigation and forecast of light frosts on ameliorated peat soils, calculation of the vapor thermolysis of organic materials, and vacuum evaporation of the liquid from a porous body. The method presented permits calculating the processes of phase transitions with variable time and coordinate steps, as well as with variable thermophysical characteristics. It was used in calculating different multifront phase transitions (liquid–gas, liquid–solid, and solid–gas).

The specific feature of the problem considered in the present paper is the dependence of the phase-transition temperature on the pressure in the system $T_{\text{ph}} = T_{\text{ph}}(P)$. The simplest way of taking into account this dependence is to hold constant the T_{ph} until complete freezing of the ith control volume and change it depending on the pressure P_{w} in the liquid phase in order to determine the freezing conditions for the next control volume. The accuracy of such a calculation will increase with decreasing calculation mesh width.

In the case of shell destruction and pressure drop in the water, a certain layer of the shell becomes supercooled due to the increase in the phase-transition temperature. In the supercooled volume, crystallization begins and the temperature increases to T_{ph}. If the water mass in the control volume is sufficient to increase its temperature to T_{ph}, then we give the temperature T_i in the control volume to be equal to T_{ph} and calculate the mass of the ice formed by formula (33). If the water mass in it is insufficient to increase its temperature to T_{ph}, then the temperature increment is calculated by the formula

$$\Delta T_i = \frac{m_{\text{e}} Q_{\text{ph}}}{c_{\text{ice}} m_{\text{ice}}} . \tag{35}$$

We perform these operations for all control volumes in the supercooled region.

When the pressure in the liquid phase exceeds the strength of the composite shell (consisting of the outer and ice shells), discontinuity and expansion of the entire system will occur. To take this into account, we have developed and realized a procedure of grid reorganization during the destruction and expansion of the system. It was taken into account that upon expansion the water should occupy a volume corresponding to the stress-free state with a density ρ = 1000 kg/m^3, and each control volume filled with ice increases by a value corresponding to the ice density equal to ρ_{iceI} = 917 kg/m^3 (it will be recalled that the mass of the control volume remains constant). To preserve the continuity of the ice interlayer, it is essential that the outer radius of the ith volume be the inner radius of the $(i + 1)$st volume.

Results and Discussion. In the calculations, we took the following characteristics of the material: T_{amb} = –30°C; T_0 = 0.5°C; R_{out} = 0.005 m; c_{w} = 4200 J/(kg·K); λ_{w} = 0.55 W/(m·K); ρ_{w} = 1000 kg/m^3; Q_{ph} = 333.6·10^3 J/kg; c_{ice} = 2100 J/(kg·K); λ_{ice} = 2.21 W/(m·K); ρ_{iceI} = 917 kg/m^3; c_{sh} = 460 J/(kg·K); λ_{sh} = 1.0 W/(m·K); ρ_{sh} = 8800 kg/m^3; E_{ice} = 9·10^9 Pa; K_{w} = 2·10^9 Pa; ν = 0.33. Upon freezing in the disperse system, a conglomerate of ice and the structuring base (material) is formed. The strength of such a system will differ from the strength of the pure ice and the pure material, and for different materials will be different. Therefore, in the calculations the total strength of the outer shell and the ice interlayer was varied over a wide range.

Consider first the case where freezing-out of water occurs in a rigid shell. Under these conditions, the whole of the filled volume of the system remains constant. However, as the water turns to ice, the volume of the latter increases, which leads to a strain of both the ice and the water and, consequently, to the appearance of stresses in the system. And in the liquid therewith a hydrostatic compression pressure is realized, and in the ice interlayer, as a result of the problem symmetry, only the normal components of stresses σ_{rr}, $\sigma_{\theta\theta} = \sigma_{\varphi\varphi}$ take place. The component σ_{rr} takes on the maximum value of P_w in magnitude on the inner surface of the ice shell adjoining the water and monotonically decreases to some value according to formula (8) on the outer surface. It has a negative value all the time, which means that the ice is compressed in the radial direction. The behavior of the components $\sigma_{\theta\theta} = \sigma_{\varphi\varphi}$ is more complicated and their spatial distribution is given by formula (9). Thus, at a small thickness of the ice interlayer they are negative and decrease in magnitude in the direction towards the inner boundary surface. As the thickness of the shell increases, $\sigma_{\theta\theta}$ and $\sigma_{\varphi\varphi}$ take on positive values near its inner surface. Thus, on the outer surface of the ice and near it the shell is compressed in the tangential direction ($\sigma_{\theta\theta} = \sigma_{\varphi\varphi} < 0$). Near the center, $\sigma_{\theta\theta}$ and $\sigma_{\varphi\varphi}$ have zero values, and, consequently strains in these directions are absent. Near the inner surface of the ice the stresses increase and on the surface itself they take maximum positive values. This means that in this region the ice shell is extended. Freezing of the system will cause an increase in the stresses to the pressures in the liquid phase $P_w = 2.1 \cdot 10^8$ Pa and a decrease in the temperature to $T = -22^oC$ at the crystallization front. When the pressure P_w is reached, ice-III will begin to form and the stresses in the system will remain constant.

The system in which the outer shell can deform and, consequently, expand behaves differently. For instance, in the above case with a rigid shell, despite the large stresses in the ice, it has no possibility to expand and, therefore, to collapse or flow. With a labile outer shell the ice has the possibility to expand and, consequently, flow and collapse. In this case, the behavior of the components of tensor stresses is described by formulas (12) and (13) and is similar, qualitatively, to the above case. Upon freezing, the stresses in the system increase and the layer consisting of ice and the outer shell, having reached critical values of the stresses, collapses, which leads to a decrease in the stresses in the system. Then the whole process described above will be repeated. This will go on until the whole of the water freezes out. Thus, the calculations show that when the liquid freezes in a closed volume, large stresses exceeding the strength of the material structure can develop.

Consider now the temperature field in the body. The temperature distribution in it is monotonic, and in the plane of the phase transition there is a kink. When the shell collapses, the pressure in the material drops, the phase-transition temperature increases, and the water turns out to be supercooled, which leads to an intensive water–ice transition. Since the heat is released intensively, the temperature in the crystallization region increases jumpwise. The calculations have shown that the value of the temperature jump depends on the shell strength. The higher the strength, the larger the stresses that the water–ice system can attain and, consequently, the phase-transition temperature takes on smaller values, which leads to a larger jump upon the destruction of the shell.

On the basis of the investigations made, it may be suggested that one method for freezing organic objects is to freeze them quickly and on all sides so that closed volumes can be formed. The inhomogeneity of the structure of the disperse system during freezing can lead to the formation of closed regions with water. The subsequent freezing causes stresses in the material associated with the water expansion upon its phase transition to ice. They will be increasing until the ultimate strength of the currently existing conglomerate of the ice and the structurizing base (material) is attained. The pressure drop in the liquid phase upon destruction will lead to its supercooled state and, consequently, crystallization of the water with heat release will begin, since ice crystals are already present in the system. In so doing, the material temperature will increase stepwise. Then the process described will be repeated until the whole of the water is frozen out.

Conclusions. The paper presents physical and mathematical models of the freezing of water confined in a spherical shell. Formulas for calculating the stress-strain state of the water–ice system are given. A method for numerical calculation of the Stefan problem that takes into account the dependence of the phase-transition temperature on the pressure in the liquid is proposed. The freezing process with account for the development of stresses in the system has been described.

NOTATION

a, constant; Bi, Biot criterion; b constant; c, specific heat capacity, J/(kg·K); c_w, specific heat capacity of water, J/(kg·K); c_{ice}, specific heat capacity of ice, J/(kg·K); c_{sh}, specific capacity of the shell, J/(kg·K); E_{ice}, elastic modulus of ice, Pa; K_w, compression modulus of water, Pa; Δm, mass of the liquid that has turned to ice, kg; M_{ice}, mass of the ice shell, kg; m_{ice}, mass of ice in the control element, kg; m_e, mass of water in the control element, kg;

P, pressure, Pa; P_w, hydrostatic pressure of water, Pa; P_{cr}, critical pressure at which ice-III is formed, Pa; P_{ex}, external pressure, Pa; R_{out}, outer radius of the spherical shell, m; r, radius, m; r_i, radius of the nodal point of the ith control volume, m; r_0, initial radius, m; r_{out}, outer radius of the liquid sphere at the initial (stress-free) moment of freezing, m; $r_{w0} = r_{ph}$, location of the zone of the water–ice phase transition obtained from the solution of the Stefan problem, m; r_{ice0}, inner radius of the ice shell, m; Δr, water sphere deformation along the radius r , m; Q_{ph}, crystallization heat, J/kg; T, temperature, ^oC; T_i, temperature of the ith control volume, ^oC; T_w, temperature of water, ^oC; T_{ice}, temperature of ice, ^oC; T_{amb}, ambient temperature, ^oC; T_0, initial temperature of the system, ^oC; T_s, temperature of the outer surface of the shell, ^oC; T_{ph}, temperature of the water–ice transition, ^oC; T_c, temperature at the center of the sphere, ^oC; t, time, sec; u, displacements along the coordinate r, m; u_{ice0}, displacements of the ice interlayer at the point r_{ice0} along the coordinate r, m; V, volume, m^3; V_0, initial volume, m^3; α, heat-transfer coefficient, $W/(m^2 \cdot K)$; ε_{rr}, $\varepsilon_{\theta\theta}$, $\varepsilon_{\varphi\varphi}$, strain-tensor components; λ, heat conductivity, $W/(m \cdot K)$; λ_w, heat conductivity of water, $W/(m \cdot K)$; λ_{ice}, heat conductivity of ice, $W/(m \cdot K)$; λ_{sh}, heat conductivity of the shell, $W/(m \cdot K)$; ν, Poisson coefficient of ice; ρ, density, kg/m^3; ρ_w, water density, kg/m^3; ρ_{ice-I}, ice-I density, kg/m^3; ρ_{sh}, shell density, kg/m^3; σ_{rr}, $\sigma_{\theta\theta}$, $\sigma_{\varphi\varphi}$, stress-tensor components, Pa. Subscripts: i, control-volume number; k, time-step number; w, water; ph; phase transition; ice, ice; out, outer radius; ex, external pressure; s, surface; amb, ambient; 0, initial; sh, shell; e, element; cr, critical; c, center.

REFERENCES

1. L. D. Landau and E. M. Lifshits, *Elasticity Theory* [in Russian], Nauka, Moscow (1987).
2. L. D. Landau and E. M. Lifshits, *Statistical Physics* [in Russian], Nauka, Moscow (1976).
3. I. P. Bazarov, *Thermodynamics* [in Russian], Vysshaya Shkola, Moscow (1976).
4. V. E. Chizhov, On the thermodynamic properties and thermal equations of state of high-pressure phases of ice, *Prikl. Mekh. Tekh. Fiz.*, No. 2, 113–123 (1993).
5. G. P. Brovka and V. A. Sychevskii, Modeling of the heat and humidity conditions in the upper layer of peat soils with account for local climatic conditions, *Prirodnye Resursy*, No. 2, 20–25 (1998).
6. G. P. Brovka, I. V. Dedyulya, E. N. Rovdan, and V. A. Sychevskii, Complex investigation and modeling of interrelated processes of the transfer of heat, moisture, and water-soluble compounds in soils, in: *Proc. IV Minsk Int. Forum "Heat and Mass Transfer–MIF-2000"* [in Russian], 22–26 May 2000, Vol 8, Minsk (2000), pp. 135–144.
7. G. P. Brovka, I. V. Dedyulya, and V. A. Sychevskii, Investigation and forecast of frosts on ameliorated peat soils, *Prirodnye Resursy*, No. 1, 13–19 (2000).
8. G. P. Brovka, I. V. Dedyulya, and V. A. Sychevskii, Simulation of thermal and moisture-content conditions of the upper layer of peat soils, *Inzh.-Fiz. Zh.*, **73**, No. 5, 999–1005 (2000).
9. G. P. Brovka, I. V. Dedyulya, E. N. Rovdan, and V. A. Sychevskiy, Heat, moisture and water-soluble compounds transfer mathematical simulation in peat grounds, *Special volume of ACTA AGROPHYSICA "Degradation of Peat Soils and Differentiation of Habitant Conditions of Hydrogenic Areas,"* Poland, Lublin (2000), pp. 121–136.
10. D. V. Aristarkhov and V. A. Sychevskii, Vapor thermolysis of rubber waste, *Vestsi Nats. Akad. Navuk Belarusi, Ser. Fiz.-Te´kh. Navuk*, No. 1, 102–108 (2002).
11. G. I. Zhuravskii and V. A. Sychevskii, Calculation of the thermolysis of rubber-tire waste, in: O. G. Martynenko (Ed.), *Heat and Mass Transfer-2001* [in Russian], Minsk (2000), pp. 13–20.
12. G. I. Zhuravskii and V. A. Sychevskii, Numerical calculation of vapor thermolysis of organic wastes, *Inzh.-Fiz. Zh.*, **76**, No. 6, 104–109 (2003).
13. V. A. Sychevskii, N. V. Pavlyukevich, L. L. Vasil'ev, Jr., and S. Rozin, Mathematical simulation of heat and mass transfer in substance evaporation from a porous body into vacuum, *Inzh.-Fiz. Zh.*, **76**, No. 1, 17–24 (2003).
14. V. A. Sychevskii, N. V. Pavlyukevich, L. L. Vasil'ev, Jr., and S. Rozin, Mathematical simulation of heat and mass transfer in a porous body in the double liquid–ice–vapor phase transition, in: *Proc. 2nd Int. Sci.-Prac. Conf. "Contemporary Energy Saving Thermal Technologies* (*Drying and Thermal Processes*)*" — SETT–2005* [in Russian], Vol. 1, 11–14 October 2005, Moscow (2005), pp. 82–85.